SIMPLIFY

1 $\dfrac{a^2 - ab}{3a} \div \dfrac{a-b}{15b^2} = \dfrac{a \cdot (a-b)}{3a} \cdot \dfrac{15\,b^2}{a-b} =$

$= \dfrac{15}{3}\, b^2 = \boxed{5\,b^2}$

2 $\dfrac{x^2 - y^2}{x^2} \cdot \dfrac{y^3}{x-y} = \dfrac{(x+y)(x-y)}{x^2} \cdot \dfrac{y^3}{x-y} =$

$= \boxed{\dfrac{(x+y)\ y^3}{x^2}}$

3 $\dfrac{\left(\dfrac{x^2-1}{x^2-3x-10}\right)}{\dfrac{x^2-3x+2}{x^2-12x+35}} = \dfrac{\dfrac{(x+1)(x-1)}{(x+2)(x-5)}}{\dfrac{(x+3)(x-1)}{(x-7)(x-5)}} =$

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How to quickly decompose a quadratic polynomial ?

$x^2 + ax + b \longrightarrow$ find 2 numbers : m , n such that :

$$\boxed{\begin{array}{l} m \cdot n = b \\ m + n = a \end{array}}$$

</div>

$= \dfrac{(x+1)(x-1)}{(x+2)(x-5)} \cdot \dfrac{(x-7)(x-5)}{(x+3)(x-1)} = \boxed{\dfrac{(x+1)(x-7)}{(x+2)(x+3)}}$

4 $\dfrac{x-2}{x-1} - \dfrac{6}{7x-7} = \dfrac{x-2}{x-1} - \dfrac{6}{7}\dfrac{1}{x-1} =$

$= \dfrac{x - 2 - 6/7}{x-1} = \boxed{\dfrac{x - 20/7}{x-1}}$

5 $\dfrac{x}{x^2-9} + \dfrac{1}{2x+6} = \dfrac{x}{(x+3)(x-3)} + \dfrac{1}{2}\dfrac{1}{x+3} =$

$= \dfrac{x + \frac{1}{2}(x-3)}{(x+3)(x-3)} = \dfrac{\frac{3}{2}x - \frac{3}{2}}{(x+3)(x-3)} = \boxed{\dfrac{3}{2}\ \dfrac{x-1}{x^2-9}}$
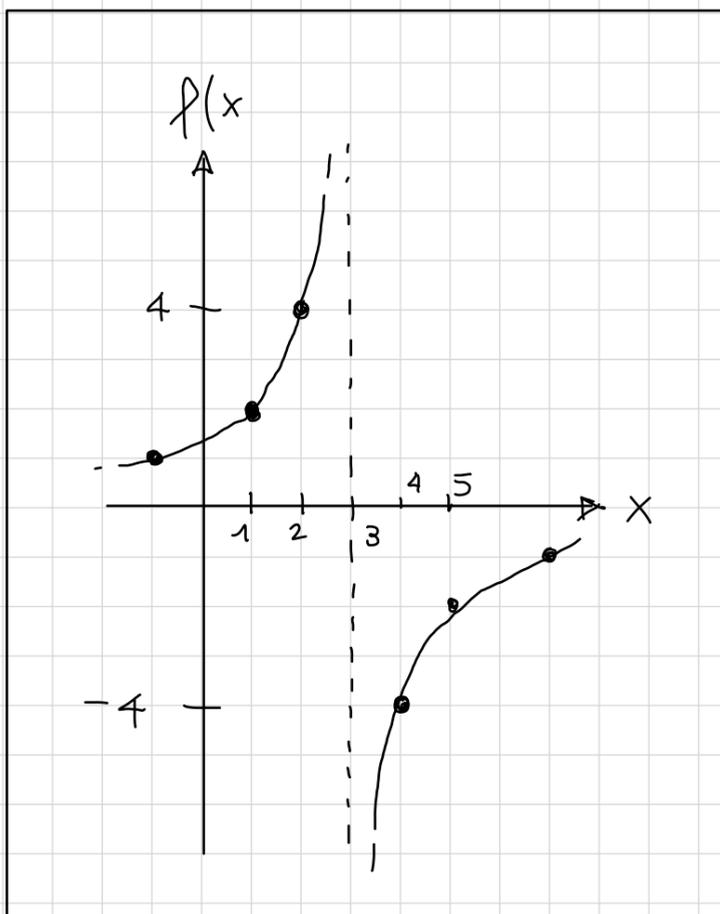
GRAPH

6 $\dfrac{-4}{x-3}$ $\longrightarrow$

division by $x-3$
$\Updownarrow$
vertical asymptote
in $x = 3$

Calculate $f(x)$ in some
points to help drawing
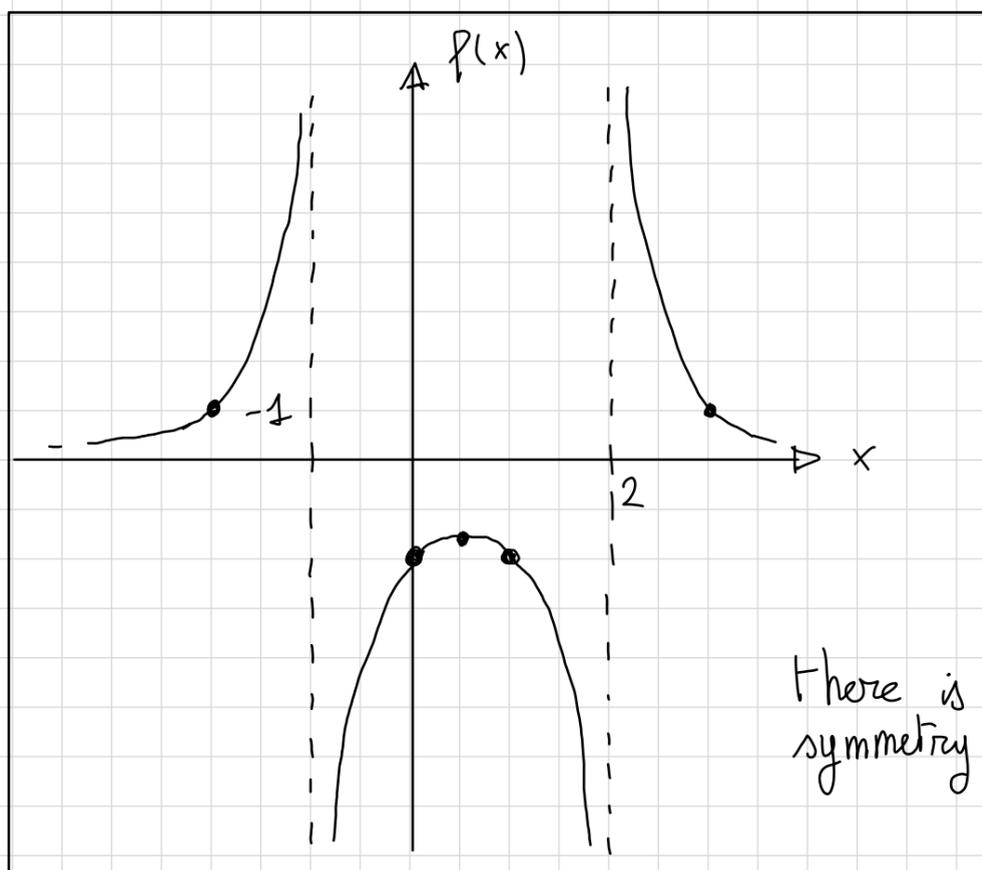
7 $\dfrac{2}{(x-2)(x+1)}$

there is
symmetry

2 vertical asymptotes

$$f(1) = \frac{2}{-1 \cdot 2} = -1$$

$$f(0) = \frac{2}{-2 \cdot 1} = -1$$

$$f(0.5) = \frac{2}{-1.5 \cdot 1.5} \approx 0.89$$

$$f(-2) = \frac{2}{-4 \cdot -1} = \frac{1}{2}$$

SOLVE

8 $\dfrac{2}{x-1} = 4 - \dfrac{x}{x-1}$;   multiply by $x-1$ AND   add the
                                             condition $x \neq 1$ (to avoid
     $2 = 4(x-1) - x$;   division by $\emptyset$)

     $0 = -2 + 4x - 4 - x$;

     $3x - 6 = 0$;

     $\boxed{x = 2}$

Check : $\dfrac{2}{2-1} = 4 - \dfrac{2}{2-1}$ ✗ ;

                $\dfrac{2}{1} = 4 - \dfrac{2}{1}$; $2 = 4 - 2$; $2 = 2$ ✓

9   $5 + \dfrac{3}{t} > -\dfrac{2}{t}$ (multiply by t ; $t \neq 0$)

     $5t + 3 > -2$; $5t > -5$; $t > -1$

   sol : $\boxed{t > -1 \quad \wedge \quad t \neq 0}$

Check : $5 + \dfrac{3}{-1} = \dfrac{-2}{-1}$; $5 - 3 = 2$ ✓

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$$\frac{x-3}{2x} = \frac{x-2}{2x+1} - \frac{1}{2} \quad ;$$

$$\frac{(x-3)(2x+1) - (x-2)(2x) + \frac{1}{2} \cdot 2x(2x+1)}{(2x)(2x+1)} = 0 \quad ;$$

$$\frac{2x^2 - 3 - 5x - 2x^2 + 4x + 2x^2 + x}{4x^2 + 2x} = 0 \quad ;$$

$$\frac{2x^2 - 3}{4x^2 + 2x} = 0 \quad ; \quad \boxed{x = \overset{+}{-}\sqrt{\frac{3}{2}}}$$

$$x \neq 0 \; ; \quad x \neq -\frac{1}{2}$$

check :

$$\frac{1}{2} - \frac{3}{2\sqrt{3/2}} = \frac{\sqrt{\frac{3}{2}} - 2}{2\sqrt{\frac{3}{2}} + 1} - \frac{1}{2} \quad ;$$

$$1 - \sqrt{\frac{8}{2}} = \frac{\sqrt{3/2} - 2}{2\sqrt{\frac{3}{2}} + 1} \quad ;$$

$$1 = \frac{\sqrt{\frac{3}{2}} - 2}{2\sqrt{\frac{3}{2}} + 1} + \sqrt{\frac{8}{2}} = \frac{\sqrt{\frac{3}{2}} - 2 + \sqrt{\frac{3}{2}} + 3}{2\sqrt{\frac{3}{2}} + 1} =$$

$$= \frac{2\sqrt{\frac{3}{2}} + 1}{2\sqrt{\frac{3}{2}} + 1} = 1 \quad \checkmark$$